International Conference "Modern aspects of singularity theory", dedicated to the 75th birthday of Sabir M. Gusein-Zade
Place: Faculty of Mathematics, room 427
Conference program

Monday, 16 June

10:00 – 10:50	A. N. Varchenko "Differential KZ equations in characteristic p"
11:00 – 11:50	F. Delgado "On Poincaré series of valuations on subfields of complex numbers"
11:50 – 13:00	*Lunch*
13:00 – 13:50	S. K. Lando "Weight systems associated to Lie algebras"
13:50 – 14:10	*Coffee break*
14:10 – 15:00	A. Szenes (online) TBA
15:10 – 16:00	I. A. Taimanov "Dynamical systems on central extensions of Lie algebras and symplectic nilmanifolds"

Tuesday, 17 June

10:00 – 10:50	M. E. Kazarian "xy swap duality in topological recursion"
11:00 – 11:50	A. Campillo (online) "Laurent series and Poincaré series"
11:50 – 13:00	*Lunch*
13:00 – 13:50	G. B. Mikhalkin "Wave front propagation and caustics in the tropical plane"
13:50 – 14:10	*Coffee break*
14:10 – 15:00	V. V. Goryunov (online) "Second Arnold-Maslov classes and some loops in the spaces of Legendrian and Lagrangian maps of surfaces"
15:10 – 16:00	A. G. Kuznetsov "Categorical resolutions of singularities"

Wednesday, 18 June

10:00 – 10:50	V. M. Buchstaber "n-valued groups"
11:00 – 11:50	E. I. Shustin (online) "Topology of expressive curves"
11:50 – 13:00	*Lunch*
13:00 – 13:50	I. Luengo TBA
14:30	*Buffet (at the dining hall)*

Thursday, 19 June

10:00 – 10:50	H. Fan TBA
11:00 – 11:50	A. P. Veselov (online) "Differential algebra of polytopes and inversion formulas"
11:50 – 13:00	*Lunch*
13:00 – 13:50	A. Y. Vesnin "Invariants of spatial graphs and corresponding knots and links"
13:50 – 14:10	*Coffee break*
14:10 – 15:00	V. V. Fock (online) "Singularities and clusters"
15:10 – 16:00	A. G. Khovanskii TBA

Friday, 20 June

10:00 – 10:50	S. O. Gorchinskiy "Powers structures on Grothendieck rings of varieties"
11:00 – 11:50	A. A. Davydov TBA
11:50 – 13:00	*Lunch*
13:00 – 13:50	E. A. Gorsky (online) "Compactified Jacobians and generic curves"
13:50 – 14:10	*Coffee break*

Abstracts

V. M. Buchstaber (Moscow, Russia)

n-valued groups

In 1971, V. M. Buchstaber and S.P. Novikov proposed a construction motivated by the theory of characteristic classes of quaternion vector bundles. This construction describes a multiplication in which the product of any pair of points is a multiset of n points. An axiomatic definition of n-valued groups, the results of their algebraic theory, and topological applications were obtained in a subsequent series of works by V. M. Buchstaber. Currently, the theory of n-valued (formal, finite, discrete, topological, and algebro-geometric) groups and their applications in various areas of mathematics and mathematical physics are being developed by a number of authors; The talk will present problems from various areas of mathematics, in the solution of which the results of the theory of n-valued groups played an important role.

A. Campillo (Valladolid, Spain)

Laurent series and Poincaré series

Joint work on Poincaré series with Felix Delgado and Sabir Gusein-Zade, and that of other collaborators, along last 30 years had provide surprising and fruitful contributions to singularities and combinatorics. Poincaré series are associated to multiindex filtrations and they are computed by means of calculus with Laurent series, natural integrals and analytic expansions. Analogous applications of Laurent series are also known in combinatorial geometry, for instance for describing lattice points or volumes of polyhedra. We describe such computations, analogies and expansions, focusing on some of the main applications of Poincaré series.

A. A. Davydov (Moscow, Russia)

TBA

F. Delgado (Valladolid, Spain)

On Poincaré series of valuations on subfields of complex numbers

Earlier, there was defined and computed the Poincaré series in several contexts, all of them in the complex case. In two recent papers we give definitions of the Poincaré series of a collection of valuations in the real setting (i. e., on the ring of germs of functions on a real analytic variety), compute them for the case of one curve or divisorial valuation on the plane and discuss some of their properties. Recently, we have extended the computations to the case of other subfields of the complex numbers for one curve or divisorial valuation.

The talk will focus on the case of a single valuation, pointing out the differences with the well-known case of branches over algebraically closed fields, as well as the qualitative difference between the semigroup and the Poincaré series.

The results have been obtained in collaboration with S. Gusein-Zade and A. Campillo.

H. Fan (Wuhan, China)

TBA

V. V. Fock (Strasbourg, France)

Singularities and clusters

In this talk we will describe a geometric relation between simple plane curve singularities classified by simply laced Cartan matrices and cluster varieties of finite type also classified by the simply laced Cartan matrices. We construct certain varieties of configurations of flags out of Dynkin diagrams and out of singularities and show that they coincide if the Dynkin diagram corresponds to the singularity.

E. A. Gorsky (Davis, USA)

Compactified Jacobians and generic curves

Given a plane curve singularity C, one can define its compactified Jacobian as a certain moduli space of sheaves on C. In the talk, I will define compactified Jacobians and review their properties, results and conjectures about them. In particular, I will define a class of "generic curves" and describe the homology of compactified Jacobians for these.

S. O. Gorchinskiy (Moscow, Russia)

Powers structures on Grothendieck rings of varieties

The talk is based on a work in progress with Danila Demin. In algebraic geometry the Grothendieck ring of varieties is being studied intensively, as well as its versions including the Grothendieck ring of varieties with actions of finite groups. There is a natural homomorphism between these rings and each of these rings has a natural lambda-structure defined in terms of (equivariant) symmetric powers of varieties. The homomorphism does not commute with lambda-structures. S.M. Gusein-Zade, I. Luengo, A. Melle-Hernandez introduced and studied the important notion of a power structure on a ring. Power structures are constructed from lambda-structures and different lambda-structures can lead to the same power structure. The main result is that the above homomorphism commutes with the power structures on the Grothendieck rings. This is obtained by means of a new general formula that expresses a power structure and the logarithm of a lambda-structure in terms of a combinatorics of trees.

V. V. Goryunov (Liverpool, United Kingdom)

Second Arnold-Maslov classes and some loops in the spaces of Legendrian and Lagrangian maps of surfaces

We introduce versions of the second Arnold-Maslov classes for paths in the spaces of Legendrian and Lagrangian maps of surfaces. We give an explicit construction – in terms of generating families – of loops in such spaces on which the values of the classes are the Euler characteristic of the surface.

M. E. Kazarian (Moscow, Russia)

xy swap duality in topological recursion

The topological recursion or the Chekhov-Eunard-Orantin recursion is an inductive procedure allowing one to solve in a uniform way many enumerative problems. The initial data of recursion involves two meromorphic functions on a Riemann surface denoted usually by x and y. The xy swap relations relate solutions of two topological recursions with the roles of the x and y functions swapped. The very existence of such relations implies numerous applications clarifying the nature of topological recursion, in particular: - it leads to explicit closed formulas for the resulting differentials of the recursion in many cases that avoid the inductive procedure - it allows one to extend the recursion to the case of degenerate pairs of x and y functions and to analyze the dependence of the resulting differentials on x and y functions - it explains KP integrablilty property observed in many enumerative problems The talk is based on a series joined papers with A.Alexandrov, B.Bychkov, P.Dunin-Barkowsky, S.Shadrin.

A. G. Khovanskii (Toronto, Canada)

TBA

A. G. Kuznetsov (Moscow, Russia)

Categorical resolutions of singularities

In the talk I will try to show a categorical point of view on resolutions of singularities in algebraic geometry.

S. K. Lando (Moscow, Russia)

Weight systems associated to Lie algebras

V. A. Vassiliev's theory of finite type knot invariants allows one to associate to such an invariant a function on chord diagrams, which are simple combinatorial objects, consisting of an oriented circle and a tuple of chords with pairwise distinct ends in it. Such functions are called "weight systems". According to a Kontsevich theorem, such a correspondence is essentially one-to-one: each weight system determines certain knot invariant.

In particular, a weight system can be associated to any semi-simple Lie algebra. However, already in the simplest nontrivial case, the one for the Lie algebra sl(2), computation of the values of the corresponding weight system is a computationally complicated task. This weight system is of great importance, however, since it corresponds to a famous knot invariant known as the colored Jones polynomial.

Last few years was a period of significant progress in understanding and computing Lie algebra weight systems, both for sl(2)- and gl(N)-weight system, for arbitrary N. These methods are based on an idea, due to M. Kazarian, which suggests a recurrence for gl(N)-weight system extended to permutations. The recurrence immediately leads to a construction of a universal gl-weight system taking values in the ring of polynomials $\mathbb{C}[N, C_1, C_2, C_3, \dots]$ in infinitely many variables and allowing for a specialization to gl(N)- and sl(N)-weight systems for any given value of N. A lot of new explicit formulas were obtained.

Simultaneously, Zhuoke Yang extended the construction to the Lie superalgebras gl($N|M$), and, together with M. Kazarian, to other classical series of Lie algebras. It happened that certain specializations of the universal gl-weight system lead to well-known combinatorial invariants of graphs, allowing thus to extend these invariants to permutations.

Certain integrability properties of the Lie algebra weight systems will be discussed.

The talk is based on work of M. Kazarian, the speaker, and N. Kodaneva, P. Zakorko, Zhuoke Yang, and P. Zinova.

I. Luengo (Madrid, Spain)

TBA

G. B. Mikhalkin (Geneva, Switzerland)

Wave front propagation and caustics in the tropical plane

The Eikonal equation of wave front propagation has its counterpart in the tropical geometry setting. The tropical plane does not have Euclidean metric. Instead it has integer lattice in each tangent space, and this structure defines evolution of any planar convex curve into a polygonal curve in arbitrary small time. Joint work with Mikhail Shkolnikov.

E. I. Shustin (Tel Aviv, Israel)

Topology of expressive curves

The talk is devoted to expressive curves introduced by S. Fomin and the speaker. These are real plane affine curves whose real point set provides complete information on the critical points of the defining polynomial. Expressive curves can be regarded as a global analog of real morsifications of singular points of plane curves, which were introduced and studied by S. Gusein-Zade and N. A'Campo. We discuss some results obtained jointly with S. Fomin and pose several open questions.

A. Szenes (Geneva, Switzerland)

TBA

I. A. Taimanov (Novosibirsk, Russia)

Dynamical systems on central extensions of Lie algebras and symplectic nilmanifolds

We will describe how the Euler equations on central extensions of Lie algebras are related to the Euler equations on the original, extended, algebras. We will also consider an infinite sequence of central extensions of nilpotent Lie algebras constructed from the Lie algebra of formal vector fields on the line, describe the orbits of coadjoint representations for these algebras, and, using compact nilmanifolds constructed from these algebras by I.K. Babenko and the author, show that covering Lie groups for symplectic nilmanifolds can have any rank as solvable Lie groups.

A. N. Varchenko (Chapel Hill, USA)

Differential KZ equations in characteristic p

The Knizhnik - Zamolodchikov (KZ) equations form a system of linear partial differential equations that exhibit profound connections with theoretical physics, representation theory, algebraic geometry, and integrable systems. In 1984, Vladimir Knizhnik and Alexander Zamolodchikov first introduced these equations within the context of conformal field theory to depict the behavior of correlation functions of vertex operators in two-dimensional quantum field theories. Since then, the equations have emerged as a central object in both mathematical physics and pure mathematics, bridging seemingly disparate fields such as Lie algebras, braid groups, hypergeometric integrals, congruences in number theory, and geometric topology. In this talk I will review properties of KZ equations in characteristic p.

A. P. Veselov (Loughborough, United Kingdom)

Differential algebra of polytopes and inversion formulas

There is a remarkable relation, due to Loday, between Lagrange inversion problem for formal series and combinatorics of Stasheff polytopes, also known as associahedra. Analogous link exists for the multiplicative inverse of the power series and special polytopes known as permutohedra.

I will explain how to prove these facts using the differential algebra of polytopes, following joint work with V.M. Buchstaber.

I will also discuss the link with the integrable PDEs and Deligne-Mumford moduli spaces in relation with the works of Witten, Kontsevich, Getzler and McMullen.

A. Y. Vesnin (Novosibirsk, Russia)

Invariants of spatial graphs and corresponding knots and links

Spatial graphs are embeddings of graphs in three-dimensional space. Therefore, the study of spatial graphs employs combinatorial and topological methods. Two spatial graphs are considered equivalent if they can be transformed into each other via an ambient isotopy of space. Since any cycle in a graph is embedded as a knot in space, the theory of spatial graphs naturally generalizes knot theory.

In this talk, we will present the relationship between Yamada polynomial invariants of certain classes of spatial graphs and Jones polynomials of their constituent knots and associated links.