STUDY SEMINAR ON ABSTRACT HOMOTOPY THEORY AND APPLICATIONS. A PROGRAM

SCIENTIFIC ADVISOR OF THE SEMINAR: PROF. D.KALEDIN
IUM, SPRING 2022 Г.
ORGANIZERS: MISHA KORNEV, GRISHA TAROYAN, LIZA ZHURAVLEVA

Seminar is intended as bipartite introduction to homotopical geometry. The two main topics will be studied simultaneously on different weekdays.

0. **(Early February 2022) Monoidal model categories** [2, Chapter IV].

I. **Homotopical categories**

I.α **(February–March 2022)** Homotopy colimits revisited [5, Part I].
- Kan extensions.
- Derived functors.
- Enriched category theory.
- (Co-)Bar construction.
- Homotopy (co-)limits — new perspective.

I.β **(April 2022)** Enriched homotopy theory [5, Part II].
- Weighted (co-)limits.
- Application to computations of homotopy (co-)limits.
- Weighted homotopy (co-)limits.
- Derived enrichment

I.γ **(May 2022)** Quasi-categories [5, Part IV], kerodon Part I, [1, Part 3].
- Models for ∞-categories.
- Simplicial categories and homotopy coherence.
- Isomorphisms in quasi-categories.

II. **Topos theory**

II.α **(February 2022)** Categorical preliminaries [4, Chapter I].
- Subobject classifiers.
- Subfunctors and Sieves.
- Heyting algebras.

II.β **(February 2022)** Sheaves in the classical sense. Review [4, Chapter II].

II.γ **(March–April 2022)** Grothendieck topologies on model categories [4, Chapter III], [6], [3, §0.3, §3].
- Grothendieck topologies on a category. (Model) Sites.
- Sheaves of sets on a site.
- Prestacks on model sites
- Local model structure. Stacks on model sites.

II.δ **(April–May 2022)** Topoi [4, Chapter IV], [6], [3, Chapter 4].
- Classical (lower) topoi.
- Geometric morphisms of topoi.
- Model topoi.

References

[1] Denis-Charles Cisinski. *Higher categories and homotopical algebra*. Vol. 180. Cambridge University Press, 2019.
[2] Mark Hovey. *Model categories*. 63. American Mathematical Soc., 2007.
[3] Peter T Johnstone. *Topos theory*. Courier Corporation, 2014.
[4] Saunders MacLane and Ieke Moerdijk. *Sheaves in geometry and logic: A first introduction to topos theory*. Springer Science & Business Media, 2012.
[5] Emily Riehl. *Categorical homotopy theory*. Vol. 24. Cambridge University Press, 2014.
[6] Bertrand Toën and Gabriele Vezzosi. "Homotopical algebraic geometry I: Topos theory". In: *Advances in mathematics* 193.2 (2005), pp. 257–372.